

IMPROVING TEACHER QUALITY PROGRAM
Mathematics Within: Algebraic Processes and Its Connections to Geometry

Participant Name: Amy TerEick

Broad Topic: Permutations

Subtopics: Patterns, factorials, probability, data organization.

Grade Level: 4th or 5th grade

Time Frame: 30 minutes

Aim:

By investigating ways to arrange a specific number of objects, students will observe a pattern that they can use to determine how many ways there are to arrange any number of objects.

Specific Objective(s):

- Using manipulatives, students will determine how many different ways there are to arrange 3 and 4 objects.
- Students will organize and record their arrangements.
- By investigating the pattern generated by 3 and 4 objects, students will predict how many ways 5 objects (as well as any number of objects) can be arranged.

Materials/Supplies:

- Graph paper
- Learning Logs
- Crayons, colored pencils, or markers
- Colored cubes or Unifix cubes (at least 5 different colors)

Lesson:

- **Introduction**

This is a brief introductory lesson to the concept of arranging objects and organizing information. Students will observe that the number of ways to arrange any number of objects (n) is $n!$ Factorials will briefly be introduced. This lesson could then be extended to a unit on probability.

- **Body**

1. Discuss with students what happens when they line up (for lunch, specialists, a drink, etc.). Ask if anyone can predict how many different ways the whole class could line up. Record suggestions on chart paper, chalkboard, or overhead.
2. Invite 1 student to the front of the room and determine as a class how many ways that students can be arranged in a line. Record arrangement. See Figure 1 (p. 3).
3. Call another student up and determine the number of ways the 2 can be arranged in a line and record on board. See Figure 1.
4. Call a third student to the front and develop all the possible different arrangements and record them on the board. See Figure 1. At this point, discuss with students a better way to record the arrangements so you don't always have to write each

- student's full name. For example, you could list the students' first initials. See Figure 2 (p. 4).
5. Once all the different ways to arrange the three students has been determined, ask students how many ways three colors could be arranged. Some may suggest the same number of ways that there were for the three students. Suggest that they prove it.
 6. In pairs, students will find and record on graph paper all the ways 3 colors can be arranged. See Figure 4 (p. 6).
 7. Once students have found and recorded all the ways 3 colors can be arranged, have them try to determine how many ways 4 colors can be arranged.
 8. Students should then look for patterns and record their observations in their learning logs. See Figure 3 (p.5) for questions.
 9. As a whole class discuss what they observe – patterns, how they arranged their colors, etc.

- **Close**

Once you've discussed with students their observations, ask if anyone can predict how many ways there are to arrange 5 objects, 6 objects, or any number of objects. Briefly introduce the concept and notation for factorial. Discuss some probabilities based on the arrangements students made. For example, if you had 4 colors (red, orange, green, and blue) arranged in any order, what's the probability that the first color is orange.

- **Application/assessment**

Students will be assessed based on the information they've recorded in their learning logs, informal observation as students are working, and by collecting students' graph paper.

Figure 1:

1 person

Bob

1 way

2 people

Bob – Gail

Gail – Bob

2 ways

3 people

Bob – Gail - Ralph

Gail – Bob – Ralph

6 ways

Bob – Ralph – Gail

Gail – Ralph – Bob

Ralph – Bob – Gail

Ralph – Gail – Bob

Figure 2:

1 person

Bob	B	1 way
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2 people

Bob – Gail	B – G	2 ways
Gail – Bob	G – B	

3 people

Bob – Gail - Ralph	B – G – R	6 ways
Gail – Bob – Ralph	G – B – R	

Bob – Ralph – Gail	B – R – G
Gail – Ralph – Bob	G – R – B

Ralph – Bob – Gail	R – B – G
Ralph – Gail – Bob	R – G – B

Figure 3:

Learning Logs

Do you see any patterns?

How do you know that you have found all the possible arrangements?

How did you find all the arrangements?

Can you predict how many ways there are to arrange 5 objects/colors?

How many different ways can all the students in the class line up?

Figure 4:

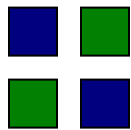
1 color

1 way



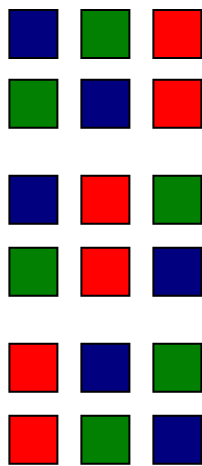
2 colors

2 ways



3 colors

6 ways



4 colors

24 ways

