

# Improving Teacher Quality Program

*Mathematics Within: Algebraic Processes and Its Connections to Geometry*

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**Broad Topic:** Connecting Algebraic Reasoning to Models

**Grade Level:** 5-6

**Time Frame:** ? minutes

## Specific Objectives:

- Relate a model to an algebraic equation
- Relate the equation back to the model

## Materials/Supplies:

- Scissors
- string
- Pencil
- paper
- Blocks
- overhead base 10 blocks

**Background information:** Patterns, variables, multiplication, arrays, rows, columns, area, perimeter

## Introduction:

Review patterns (patterns repeat, they can help you predict, they help you find mistakes). Review how patterns can be described (don't start with "it", not just list of numbers).

## Body:

### Part I

*(Do in partners, one is cutter and one is recorder)*

**Directions:** Take a fairly long piece of string (12 inches) and drape it over your finger. You have one piece of string. You have 2 ends draping over. Take one long cut through both of the ends. You now have made one cut and have 3 pieces of string. Make another cut. How many total pieces of string do you have now (5)? Let us organize the information in a table to try to find some connection between number of cuts and number of pieces of string. Get as far as you can in the next few minutes.

Number of cuts	Total number of pieces of string
0	1
1	3
2	5
3	7
4	9
5	11
6	13

What patterns do you see?

*(Welcome all statements of pattern. Make sure that they see that the number of pieces of string "grows" by 2 each time and that the difference between the number of cuts and the total number of pieces of string increases by 1 each time. What would you get if you kept a running total of the total number of pieces of string?)*

How could you predict the total number of pieces of string for any number of cuts? I'll call the number of cuts "n."

*(The total number of pieces =  $2n + 1$ )*

Why? How does this equation we made relate to the model? Write this down and save it in class notes. It's an example of what I mean when I ask you to describe what the equation means:

***"The number of pieces of string =  $2n + 1$ " means that you get 2 new pieces of string for every new cut you make and you still have the original piece of string over your finger.***

Extensions: Use 'number of cuts' as the x coordinate and 'total pieces of string' as the y coordinate. What do you get if you graph the equation? Take a piece of paper and fold it once. How many regions do you get? Fold it again. How many regions do you get? Make a table. Find the patterns. Make a graph.

Number of folds	Number of regions
1	2
2	4
3	8

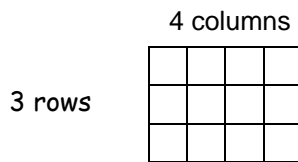
*(This is a good introduction to the CMP unit Variables and Patterns and an examination of the difference between linear, quadratic, and exponential equations. Thank you, Kathleen Kramer.)*

$Y = 25x + 100$  (\$25 a bike + \$100 down)  
 $Y = 50x$  (\$50 a bike)  
 Intersects at 4 bikes.

### Part II

Now we will look at a different model and try to come up with an equation (Overhead base 10 blocks and blocks).

### Review arrays (area model)



How did you know there would be 12 blocks? What is the perimeter of this rectangle? (review) If you added a row, what would the new area be? What can you tell me about the number 16? What would the new perimeter be?

If I made a new rectangle with 2 rows and 8 columns, what would the area be? What would the perimeter be? How could I get the biggest possible perimeter for an area of 16 sq. units? (review) Why does this happen?

Pick your favorite 2-digit number (make sure your favorite is not the same as your neighbor's favorite). You are going to multiply the number by itself without using calculators. Before you multiply it, what can you tell me about your product?

After you multiply your number, take one away from the top number (the number of rows) and add one to the bottom number (the number of columns) and multiply. Continue to do this (take one away from the top number and add one to the bottom number), finding the products, until I tell you to stop.

Questions: Did you have to keep multiplying? Why? Check your patterns with your neighbor's. Did you get the same ones? Describe your patterns. You have described similar patterns. How could you show what happens with a model? You have blocks out. How could the blocks help?

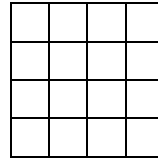
*(Accept models that make sense. Basically:*

*You take off a row and make it a column. When you do that you have one left over the first time. Do it again and you have 3 left over. If you combine the leftover blocks, you have consecutive square numbers.)*

A simpler example would be if I start with:

$$\begin{array}{r} 4 \\ \times 4 \\ \hline 16 \end{array}$$

(This is A) (This is also A) (This is A<sup>2</sup>)

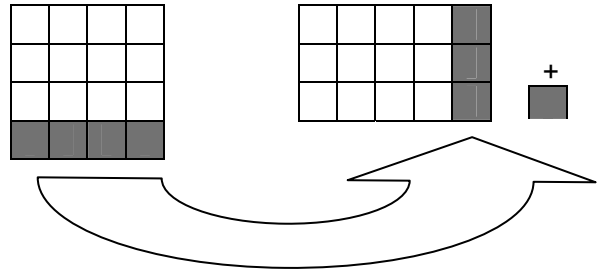


The next step is:

$$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$$

(This is A) (This is also A) (This is A<sup>2</sup>)

What do we call 3 in terms of A? What do we call 5?

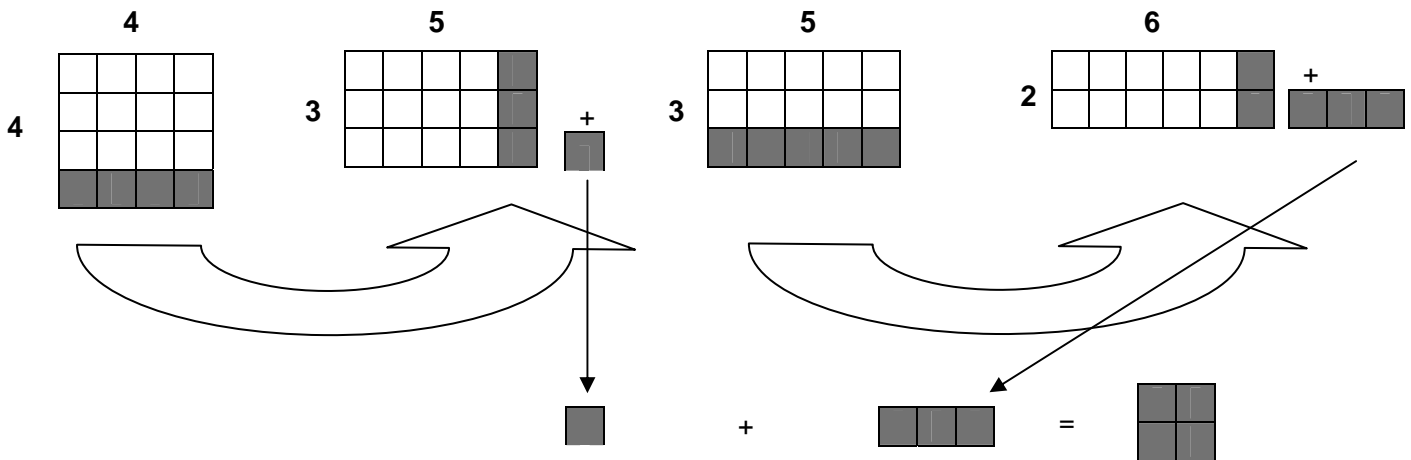


So, we have  $(A-1)(A+1) = 15$ .

Refer to the model we made. In the model, what do you do to get the answer? How can we describe the answer of 15 in terms of the model?

*(Make a square with blocks. You start with a square. Take a row and make it a column. You have one less row and one more column, and one block leftover. Your answer is the area of the square minus one block.)*

Try this again with  $(A-2)(A+2) = 12$



**Next lesson:**

Go into depth with the area model. Review the area model of multiplication. Show the above model with the area model (multiplication of negative numbers will be involved). This will show the distributive property. Find a general equation for any number (B) less than or greater than the square. Relate the equation to the model.  $(A-B)(A+B) = (A^2 - B^2)$

**Evaluation:**The staircase problem

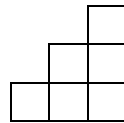
These are the staircases. The number below each one is the "term" number. How can you predict the total number of blocks needed for any staircase given only the term number (N)? Show me a model for how you figured this out. Give me an equation for your model and describe how each part of your equation fits the model.



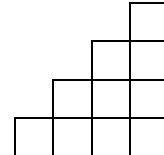
Term: 1



2



3



4

Extra: Do the same for the perimeter of each staircase.